APS Homework 4: Dynamic Programming

Problem 1: Coin Change

You are a cashier in a country that has coin denominations C. For example, in the USA, the denominations would be C = {1, 5, 10, 25, 50, 100} for {penny, nickel, dime, quarter, half-dollar, and dollar}. Given a positive integer x representing the amount of change you need to give a customer, you want to give the customer the smallest number of coins that add up to x.

Problem 1a: Describe a Dynamic Programming algorithm for finding the smallest number of coins in C such that their values add up to x.

Problem 1b: Prove that the algorithm you provided in *Problem 1a* is correct for any arbitrary set of positive integer coin denominations C and any arbitrary positive integer x.

Problem 2: Road Trip

You are about to embark on a road trip that is x miles long in a car whose gas tank can hold g gallons of gas and which can travel m miles per gallon. The route is a straight path, and there are n gas stations along the route. Let d_i denote the distance between the starting point and gas station i, and let p_i denote the per-gallon price of gas at gas station i. Determine which gas stations you should stop at *and* how much gas you must buy at each station such that you can reach your destination without running out of gas along the way while minimizing the amount of money you spent. You can run out of gas when you reach the destination (i.e., when you travel the full x miles) or the moment you reach a gas station, but you cannot run out of gas while you are driving.

Problem 2a: Describe a Dynamic Programming algorithm for determining which gas stations you need to stop at to travel the full x miles while minimizing the amount of money you spend on gas. Assume g, m, x, d_i, and p_i are all positive numbers (but not necessarily integers).

Problem 2b: Prove that the algorithm you provided in *Problem 2a* is correct for any arbitrary gas tank capacity g, any arbitrary gas mileage m, any arbitrary route distance x, and any arbitrary set of gas stations with any arbitrary values of d_i and p_i.